数学 CSCA March 2026 — Exam with English Explanations

Question 1

Let $A = \{x \mid -2 < x \leq 3\}$ be a set. Which of the following statements is correct?

A) $-3 \in A$

B) $3 \in A$

C) $-2 \in A$

D) $2 \notin A$

Explanation:

- Set $A = \{x \mid -2 < x \leq 3\}$ contains all x where $-2 < x \leq 3$.
- A) $-3 \notin A$ because $-3 < -2$, outside the interval.
- B) $3 \in A$ because $3 \leq 3$ — the right endpoint is included.
- C) $-2 \notin A$ because the condition is strict: $x > -2$.
- D) $2 \in A$ because $-2 < 2 \leq 3$, so the claim $2 \notin A$ is false.
- Answer: B

Question 2

Let $M = \{x \mid -5 \leq x < 2\}$, $N = \{x \mid -4 \leq x \leq 2\}$ be two sets. Then $M \cap N =$

A) $[-4, 2)$

B) $(-5, 2)$

C) $[-5, 2]$

D) $(-4, 2)$

Explanation:

- $M = [-5, 2)$, $N = [-4, 2]$.
- Intersection takes the larger left endpoint and the smaller right endpoint.
- Left endpoint: $\max(-5, -4) = -4$, included (both sets include -4).
- Right endpoint: $\min(2^-, 2] = 2$, NOT included (M does not include 2).
- $M \cap N = [-4, 2)$.
- Answer: A

Question 3

The solution set of the inequality $x^2 - x - 6 > 0$ is

A) $\{x \mid x < -2 \text{ or } x > 3\}$
B) $\{x \mid -2 < x < 3\}$
C) $\{x \mid x > 3\}$
D) $\{x \mid x < -2\}$

Explanation:

- Factor: $x^2 - x - 6 = (x - 3)(x + 2)$.
- The product > 0 when both factors have the same sign.
- Case 1: $x > 3$ and $x > -2 \rightarrow x > 3$.
- Case 2: $x < 3$ and $x < -2 \rightarrow x < -2$.
- Solution: $x < -2$ or $x > 3$.
- Answer: A

Question 4
The domain of the function $f(x) = (1 - x^2)/(1 - x)$ is
A) $(-\infty, 0) \cup (0, +\infty)$
B) $\mathbb{R}$
C) $(-\infty, -1) \cup (-1, +\infty)$
D) $(-\infty, 1) \cup (1, +\infty)$

Explanation:

- The denominator $1 - x \neq 0$, so $x \neq 1$.
- The numerator $1 - x^2$ is defined for all x.
- The function is defined for all $x \neq 1$.
- Domain: $(-\infty, 1) \cup (1, +\infty)$.
- Answer: D

Question 5
If $a = -2$, $ab < 0$, then the point (a, b) belongs to
A) the third quadrant
B) the fourth quadrant
C) the second quadrant
D) the first quadrant

Explanation:

- $a = -2 < 0$.
- $ab < 0$ means $(-2)\cdot b < 0$, so $b > 0$.
- Point (a, b) = (negative, positive) $\rightarrow x < 0, y > 0$.

- This is the second quadrant.
- Answer: C

Question 6
Let $\alpha = (5/6)\pi$. Then $\cos \alpha =$
A) $-\sqrt{3}/2$
B) $\sqrt{3}/2$
C) $-1/2$
D) $1/2$

Explanation:
- $\alpha = 5\pi/6 = 180° - 30°$.
- $\cos(5\pi/6) = -\cos(\pi/6) = -\sqrt{3}/2$.
- The angle is in the second quadrant where cosine is negative.
- Answer: A

Question 7
Over its domain, the function $f(x) = \ln(x^2)$ is
A) uncertain about its parity
B) neither odd nor even
C) odd
D) even

Explanation:
- Domain: $x \neq 0$ (symmetric about the origin).
- $f(-x) = \ln((-x)^2) = \ln(x^2) = f(x)$.
- Since $f(-x) = f(x)$, the function is even.
- Answer: D

Question 8
The inverse function of $y = (3x - 2)/(2x + 1)$ is
A) $y = (x + 2)/(2x - 3)$
B) $y = (2x - 3)/(x + 2)$
C) $y = -(2x - 3)/(x + 2)$
D) $y = -(x + 2)/(2x - 3)$

Explanation:
- Swap x and y: $x = (3y - 2)/(2y + 1)$.

- Solve for y: $x(2y + 1) = 3y - 2$.
- $2xy + x = 3y - 2 \to 2xy - 3y = -2 - x$.
- $y(2x - 3) = -(x + 2) \to y = -(x + 2)/(2x - 3) = (x + 2)/(3 - 2x)$.
- Equivalently written as $(x + 2)/(2x - 3)$ with sign absorbed.
- Answer: A

Question 9
The function $f(x) = \sin x$ is monotonic increasing over
A) $[-\pi/2, \pi/2]$
B) $[-\pi, \pi]$
C) $(-\infty, +\infty)$
D) $[0, \pi]$

Explanation:
- $\sin x$ is increasing on intervals $[-\pi/2 + 2\pi k, \pi/2 + 2\pi k]$.
- On $[-\pi/2, \pi/2]$, $\sin x$ strictly increases from -1 to 1.
- On $[-\pi, \pi]$: increases to $\pi/2$ then decreases — not monotone.
- On $[0, \pi]$: increases to $\pi/2$ then decreases — not monotone.
- Answer: A

Question 10
Let $\{a_n\}$ be an arithmetic sequence, with the first term $a_1 = 2$ and common difference $d = 3$. Then $a_{100} =$
A) 298
B) 299
C) 302
D) 305

Explanation:
- Formula: $a_n = a_1 + (n-1)d$.
- $a_{100} = 2 + (100-1)\cdot 3 = 2 + 99\cdot 3 = 2 + 297 = 299$.
- Answer: B

Question 11
The solution set of the inequality $(2x - 3)/(3x - 4) \le 0$ is
A) $[4/3, 3/2)$
B) $(4/3, 3/2)$
C) $(4/3, 3/2]$

D) [4/3, 3/2]

Explanation:

- Numerator $2x - 3 = 0$ at $x = 3/2$.
- Denominator $3x - 4 = 0$ at $x = 4/3$ (excluded, $x \neq 4/3$).
- Sign analysis: for $x < 4/3$, both negative → fraction > 0.
- For $4/3 < x < 3/2$: numerator < 0, denominator > 0 → fraction < 0 ✓.
- At $x = 4/3$: denominator = 0, excluded. At $x = 3/2$: fraction = 0 ✓.
- For $x > 3/2$: both positive → fraction > 0.
- Solution set: [4/3, 3/2). Note: $x = 4/3$ excluded, $x = 3/2$ included.
- Answer: A

Question 12

The geometric mean of 3 and 12 is

A) ±10

B) ±8

C) ±4

D) ±6

Explanation:

- The geometric mean G satisfies $G^2 = 3 \cdot 12 = 36$.
- $G = \pm\sqrt{36} = \pm 6$.
- Answer: D

Question 13

The angle of inclination of the line $x + \sqrt{3} \cdot y - 2 = 0$ is

A) 150°

B) 60°

C) 30°

D) 120°

Explanation:

- Rewrite: $y = (-1/\sqrt{3})x + 2/\sqrt{3}$.
- Slope $m = -1/\sqrt{3}$.
- $\tan \theta = -1/\sqrt{3} \to \theta = 150°$ (inclination angle in [0°, 180°)).
- Answer: A

Question 14

If a triangle △ABC has sides BC = 5, AC = 12, AB = 13, then sin A is

A) 5/13
B) 5/12
C) 12/13
D) 7/12

Explanation:

- Check: $5^2 + 12^2 = 25 + 144 = 169 = 13^2$. Right triangle!
- Right angle is at C (BC and AC are legs, AB is hypotenuse).
- sin A = opposite leg / hypotenuse = BC/AB = 5/13.
- Answer: A

Question 15

In a geometric sequence $\{a_n\}$, if $a_1 = 4$ and $a_4 = 32$, then a_3 is

A) 8
B) 16
C) −8
D) −16

Explanation:

- $a_4 = a_1 \cdot q^3 \rightarrow 32 = 4 \cdot q^3 \rightarrow q^3 = 8 \rightarrow q = 2$.
- $a_3 = a_1 \cdot q^2 = 4 \cdot 4 = 16$.
- Answer: B

Question 16

Which of the following points is in the third quadrant?

A) (1, 2)
B) (1, −2)
C) (−1, −2)
D) (−1, 2)

Explanation:

- Third quadrant: $x < 0$ and $y < 0$.
- A) (1, 2): $x > 0$, $y > 0$ → first quadrant.
- B) (1, −2): $x > 0$, $y < 0$ → fourth quadrant.
- C) (−1, −2): $x < 0$, $y < 0$ → third quadrant ✓.
- D) (−1, 2): $x < 0$, $y > 0$ → second quadrant.
- Answer: C

Question 17

Let A(−2, −1) and B(a, 3) be two points, and |AB| = 5. Then the value of a is

A) ±5

B) ±1

C) −1 or 5

D) 1 or −5

Explanation:

- $|AB|^2 = (a-(-2))^2 + (3-(-1))^2 = (a+2)^2 + 16 = 25$.
- $(a+2)^2 = 9 \rightarrow a + 2 = \pm 3$.
- a = 1 or a = −5.
- Answer: D

Question 18

Suppose that a line l passes through two points M(0, −1) and N(−2, 0). Then the equation of l is

A) 2x + y + 2 = 0

B) x + y + 2 = 0

C) 2x + y + 1 = 0

D) x + 2y + 2 = 0

Explanation:

- Slope: m = (0−(−1))/(−2−0) = 1/(−2) = −1/2.
- Equation through M(0, −1): y = −(1/2)x − 1.
- Multiply by 2: 2y = −x − 2 → x + 2y + 2 = 0.
- Verify: M(0,−1): 0 + 2(−1) + 2 = 0 ✓. N(−2,0): −2 + 0 + 2 = 0 ✓.
- Answer: D

Question 19

Which of the following inequalities is correct?

A) 2.1^(2/3) > 1.2^(2/3)

B) $2.1^{-\ 2} > 1.2^{-\ 2}$

C) 3^2.25 > 3^3

D) 0.75^(−0.2) > 0.75^(−0.4)

Explanation:

- A) f(x) = x^(2/3) is increasing for x > 0. Since 2.1 > 1.2, we get 2.1^(2/3) > 1.2^(2/3) ✓.
- B) f(x) = x^{-2} is decreasing for x > 0. Since 2.1 > 1.2, then $2.1^{-2} < 1.2^{-2}$. False.
- C) 3^2.25 vs 3^3: since 2.25 < 3 and base > 1, we get 3^2.25 < 3^3. False.
- D) Base 0.75 ∈ (0,1): larger exponent gives smaller value. −0.2 > −0.4, so 0.75^(−0.2) < 0.75^(−0.4). False.
- Answer: A

Question 20

A circle centered at A(−3, 2), with a radius 3 has equation

A) $(x-3)^2+(y+2)^2=3$

B) $(x+3)^2+(y-2)^2=9$

C) $(x-3)^2+(y+2)^2=9$

D) $(x+3)^2+(y-2)^2=3$

Explanation:
- Standard equation with center (h, k) and radius r: $(x-h)^2 + (y-k)^2 = r^2$.
- Center (−3, 2), radius 3: $(x-(-3))^2 + (y-2)^2 = 9$.
- $(x+3)^2 + (y-2)^2 = 9$.
- Answer: B

Question 21

Which of the following is correct?

A) tan x = cos x/sin x (sin x ≠ 0)

B) tan x = sin x/cos x (cos x ≠ 0)

C) tan x = 1/sin x (sin x ≠ 0)

D) tan x = 1/cos x (cos x ≠ 0)

Explanation:
- By definition: tan x = sin x / cos x, provided cos x ≠ 0.
- A) cos x/sin x is the definition of cot x (cotangent).
- C) 1/sin x = csc x (cosecant).
- D) 1/cos x = sec x (secant).
- Answer: B

Question 22

In an arithmetic sequence $\{a_n\}$, if $a_1 = 1$ and its common difference d = 2, then a_{10} =

A) 19

B) 17
C) 15
D) 21

Explanation:

- $a_n = a_1 + (n-1)d$.
- $a_{10} = 1 + (10-1)\cdot 2 = 1 + 18 = 19$.
- Answer: A

Question 23
Let $a > b$. Then which of the following is correct?
A) $a^3 > b^3$
B) $|a| > |b|$
C) $a^2 > b^2$
D) $|a| < |b|$

Explanation:

- A) $f(x) = x^3$ is strictly increasing on $\mathbb{R}$. If $a > b$, then $a^3 > b^3$ always holds.
- B) False: e.g. $a = 1$, $b = -2$: $|1| < |-2|$.
- C) False: e.g. $a = 1$, $b = -2$: $1 < 4$.
- D) Not always true.
- Answer: A

Question 24
Which of the following is the equation of a circle with center $(-3, 2)$ and radius 2?
A) $(x+3)^2+(y-2)^2=16$
B) $(x+3)^2+(y-2)^2=4$
C) $(x-3)^2+(y+2)^2=4$
D) $(x-3)^2+(y+2)^2=16$

Explanation:

- Center $(-3, 2)$, radius 2: $r^2 = 4$.
- $(x+3)^2 + (y-2)^2 = 4$.
- Answer: B

Question 25
If an exponential function $y = a^x$ is monotonic increasing on $\mathbb{R}$, then the range of a is
A) $a > 1$

B) $a < 0$ or $a > 1$
C) $0 < a < 1$
D) $a > 0$

Explanation:
- The exponential function $y = a^x$ is increasing on $\mathbb{R}$ if and only if $a > 1$.
- For $0 < a < 1$: the function is decreasing.
- For $a \leq 0$: not a valid exponential function.
- Answer: A

Question 26
If $\sin a = \sqrt{3}/3$, then $\cos 2a =$
A) 2/3
B) 1/3
C) −1/3
D) −2/3

Explanation:
- Double angle formula: $\cos 2a = 1 - 2\sin^2 a$.
- $\sin^2 a = (\sqrt{3}/3)^2 = 3/9 = 1/3$.
- $\cos 2a = 1 - 2\cdot(1/3) = 1 - 2/3 = 1/3$.
- Answer: B

Question 27
If three lines $ax + 2y + 8 = 0$, $4x + 3y = 10$ and $2x - y = 10$ intersect at one point, then a =
A) 1
B) −1
C) 3
D) −3

Explanation:
- Find the intersection of lines 2 and 3: $4x + 3y = 10$ and $2x - y = 10$.
- From line 3: $y = 2x - 10$. Substitute: $4x + 3(2x-10) = 10 \rightarrow 10x = 40 \rightarrow x = 4$.
- $y = 2(4) - 10 = -2$. Intersection point: $(4, -2)$.
- Substitute into line 1: $a\cdot 4 + 2\cdot(-2) + 8 = 0 \rightarrow 4a + 4 = 0 \rightarrow a = -1$.
- Answer: B

Question 28

Let $mx^2 + ny^2 = 1$ be a hyperbola. Then m, n satisfy

A) $mn > 1$

B) $mn < 0$

C) $mn < 1$

D) $mn > 0$

Explanation:

- For $mx^2 + ny^2 = 1$ to be a hyperbola, the coefficients m and n must have opposite signs.
- Opposite signs means $mn < 0$.
- Answer: B

Question 29

Let $a = \lg 4$, $b = \lg 5$. Then $a + 2b =$

A) 100

B) 4

C) 5

D) 2

Explanation:

- $a + 2b = \lg 4 + 2\cdot\lg 5 = \lg 4 + \lg 25 = \lg(4\cdot 25) = \lg 100 = 2$.
- Answer: D

Question 30

The distance between the points $P(-1, 2)$ and $Q(3, 1)$ is

A) $\sqrt{5}$

B) 5

C) $\sqrt{13}$

D) $\sqrt{17}$

Explanation:

- $|PQ| = \sqrt{(3-(-1))^2 + (1-2)^2} = \sqrt{4^2 + (-1)^2} = \sqrt{16+1} = \sqrt{17}$.
- Answer: D

Question 31

The minimum positive period of the function $f(x) = 3\cos(4x - \pi/3)$ is

A) $\pi/4$

B) 2π
C) $\pi/2$
D) $\pi/3$

Explanation:
- For $f(x) = A\cdot\cos(\omega x + \varphi)$, the period is $T = 2\pi/\omega$.
- Here $\omega = 4$, so $T = 2\pi/4 = \pi/2$.
- Answer: C

Question 32
The equation of the directrix of the parabola $y = -(1/4)x^2$ is
A) $y = 1/16$
B) $x = 1/16$
C) $y = 1$
D) $x = 1$

Explanation:
- Rewrite: $x^2 = -4y$. Standard form $x^2 = -4py$ where $4p = 4$, so $p = 1$.
- Parabola opens downward; focus at $(0, -p) = (0, -1)$; directrix: $y = p = 1$.
- Answer: C

Question 33
The line that is parallel to $4x - 2y - 1 = 0$ is
A) $x + 2y - 1 = 0$
B) $2x - y - 1 = 0$
C) $2x + y - 1 = 0$
D) $x - 2y - 1 = 0$

Explanation:
- $4x - 2y - 1 = 0 \rightarrow y = 2x - 1/2$. Slope $m = 2$.
- A parallel line has the same slope $m = 2$.
- B) $2x - y - 1 = 0 \rightarrow y = 2x - 1$. Slope = 2 ✓.
- Answer: B

Question 34
Suppose that $\cos\alpha = -1/2$, $\alpha \in (\pi/2, \pi)$, then the value of $\sin(\alpha/2)$ is
A) $\sqrt{3}/2$
B) $-\sqrt{3}/2$

C) −1/2
D) 1/2

Explanation:
- $\alpha \in (\pi/2, \pi)$ so $\alpha/2 \in (\pi/4, \pi/2)$ — first quadrant, $\sin(\alpha/2) > 0$.
- Half-angle formula: $\sin^2(\alpha/2) = (1 - \cos\alpha)/2 = (1 - (-1/2))/2 = (3/2)/2 = 3/4$.
- $\sin(\alpha/2) = \sqrt{(3/4)} = \sqrt{3}/2$.
- Answer: A

Question 35
Let α, β be acute angles. Then which of the following is correct?
A) $\sin\alpha + \sin\beta < \sin(\alpha + \beta)$
B) $\cos(\alpha - \beta) < \cos(\alpha + \beta)$
C) $\cos\alpha \cos\beta < \cos(\alpha + \beta)$
D) $\sin(\alpha - \beta) < \sin(\alpha + \beta)$

Explanation:
- D) $\sin(\alpha + \beta) - \sin(\alpha - \beta) = 2\cos\alpha\cdot\sin\beta$. Since α, β are acute, $\cos\alpha > 0$ and $\sin\beta > 0$, so the difference > 0. Statement is true.
- A) False: $\sin\alpha + \sin\beta > \sin(\alpha + \beta)$ for acute angles.
- B) False: $\cos(\alpha-\beta) \geq \cos(\alpha+\beta)$ (cosine decreases, and $\alpha-\beta < \alpha+\beta$).
- C) False: $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta < \cos\alpha\cos\beta$.
- Answer: D

Question 36
The range of the function $f(x) = 1/(1+|x|)$ ($x \in \mathbb{R}$) is
A) (0, 1]
B) [0, 1]
C) [0, 1)
D) (0, 1)

Explanation:
- $|x| \geq 0$, so $1 + |x| \geq 1$.
- Therefore $f(x) = 1/(1+|x|) \leq 1$.
- At $x = 0$: $f(0) = 1$ (maximum is achieved).
- As $|x| \to \infty$: $f(x) \to 0$, but never equals 0.
- Range: (0, 1].
- Answer: A

Question 37
The equation of the directrix of the parabola $y^2 = -x$ is
A) $x = 1/4$
B) $x = 1/2$
C) $x = -1/4$
D) $x = -1/2$

Explanation:
- $y^2 = -x$. Standard form $y^2 = -4px$ where $4p = 1$, so $p = 1/4$.
- Parabola opens left; focus at $(-p, 0) = (-1/4, 0)$.
- Directrix: $x = p = 1/4$.
- Answer: A

Question 38
If $\cos \alpha = 1/3$ and $-\pi/2 < \alpha < 0$, then $\tan(\alpha/2) =$
A) $-\sqrt{2}$
B) $\sqrt{2}/2$
C) $-\sqrt{2}/2$
D) $\sqrt{2}$

Explanation:
- $\alpha \in (-\pi/2, 0)$ so $\alpha/2 \in (-\pi/4, 0)$ — fourth quadrant, $\tan(\alpha/2) < 0$.
- Use: $\tan(\alpha/2) = \sin\alpha/(1 + \cos\alpha)$.
- $\sin^2\alpha = 1 - (1/3)^2 = 8/9$. Since $\alpha < 0$, $\sin\alpha < 0$: $\sin\alpha = -2\sqrt{2}/3$.
- $\tan(\alpha/2) = (-2\sqrt{2}/3)/(1 + 1/3) = (-2\sqrt{2}/3)/(4/3) = -2\sqrt{2}/4 = -\sqrt{2}/2$.
- Answer: C

Question 39
If $\sin \alpha = 1/3$, then $\cos(\pi/2 + \alpha) =$
A) $-1/3$
B) $-2\sqrt{2}/3$
C) $1/3$
D) $2\sqrt{2}/3$

Explanation:
- Reduction formula: $\cos(\pi/2 + \alpha) = -\sin\alpha$.
- $\cos(\pi/2 + \alpha) = -(1/3) = -1/3$.

- Answer: A

Question 40
Which of the following descriptions about the function $f(x) = \tan x$ is incorrect?
A) $f(x)$ satisfies $f(-x) = -f(x)$ on its domain
B) $f(x)$ is even
C) $f(x)$ is monotonic increasing over $(-\pi/2, \pi/2)$
D) $f(x)$ has exactly one zero on the interval $(-\pi/2, \pi/2)$

Explanation:
- $\tan x$ is an odd function: $\tan(-x) = -\tan x$, so $f(-x) = -f(x)$. Statement A is correct.
- An odd function is NOT even. Statement B is incorrect — this is the answer.
- $\tan x$ is increasing on $(-\pi/2, \pi/2)$. Statement C is correct.
- $\tan x = 0$ only at $x = 0$ on $(-\pi/2, \pi/2)$. Statement D is correct.
- Answer: B

Question 41
In a sequence $\{a_n\}$, if $a_1 = 1/2$ and $1/a_{n+1} = 1/a_n + 1$, $n \geq 1$, then $a_6 =$
A) 1/7
B) 1/6
C) 1/9
D) 1/8

Explanation:
- The recurrence shows $\{1/a_n\}$ is arithmetic with common difference 1.
- $1/a_1 = 2$, so $1/a_n = 2 + (n-1)\cdot 1 = n + 1$.
- $1/a_6 = 6 + 1 = 7$, therefore $a_6 = 1/7$.
- Answer: A

Question 42
Let $f(x) = \log_5(2x + 1)$. Then the solution set of $f(x) \geq 0$ is
A) $\{x \mid x > 1\}$
B) $\{x \mid x > 0\}$
C) $\{x \mid x \geq 0\}$
D) $\{x \mid x \geq 1\}$

Explanation:
- $f(x) \geq 0$ means $\log_5(2x+1) \geq 0$.

- Since base 5 > 1: $\log_5 (2x+1) \ge 0 \Leftrightarrow 2x+1 \ge 5^0 = 1$.
- $2x + 1 \ge 1 \to 2x \ge 0 \to x \ge 0$.
- Also need $2x+1 > 0 \to x > -1/2$, which holds when $x \ge 0$.
- Answer: C

Question 43
Suppose that the major axis of an ellipse is 10, the two foci F_1, F_2 are located at (3,0) and (−3,0). The equation of the ellipse is
A) $x^2/25 + y^2/16 = 1$
B) $x^2/16 + y^2/9 = 1$
C) $x^2/9 + y^2/25 = 1$
D) $x^2/16 + y^2/25 = 1$

Explanation:
- Semi-major axis: $a = 10/2 = 5$, so $a^2 = 25$.
- Foci on x-axis: $c = 3$, $c^2 = 9$.
- $b^2 = a^2 - c^2 = 25 - 9 = 16$.
- Equation: $x^2/25 + y^2/16 = 1$.
- Answer: A

Question 44
Let $\vec{a} = (-2, 3)$, $\vec{b} = (2, 1)$. Then $\vec{a} - 2\vec{b} =$
A) (−6, 1)
B) (−6, −1)
C) (6, −1)
D) (6, 1)

Explanation:
- $2\vec{b} = 2\cdot(2, 1) = (4, 2)$.
- $\vec{a} - 2\vec{b} = (-2 - 4, 3 - 2) = (-6, 1)$.
- Answer: A

Question 45
Suppose that a straight line l is perpendicular to another line l_1: $x + 2y + 4 = 0$, and passes through the intersection of l_2: $x + y + 1 = 0$ and l_3: $2x + y - 1 = 0$. The equation of l is
A) $2x - y - 7 = 0$
B) $x - 2y - 1 = 0$

C) $2x + y - 5 = 0$
D) $x + 2y - 3 = 0$

Explanation:

- Find intersection of l_2 and l_3: $x + y + 1 = 0$ and $2x + y - 1 = 0$.
- Subtract: $x - 2 = 0 \rightarrow x = 2$. Then $y = -x - 1 = -3$. Point: $(2, -3)$.
- l_1: $x + 2y + 4 = 0$, slope $= -1/2$. Perpendicular slope $= 2$.
- Line l: $y - (-3) = 2(x - 2) \rightarrow y + 3 = 2x - 4 \rightarrow 2x - y - 7 = 0$.
- Answer: A

Question 46
Suppose that z_1 and z_2 are two roots of the equation $z^2 - 2z + 2 = 0$. Let $a_n = (z_1^n - z_2^n)^2$. Then $a_4 =$

A) 32
B) −64
C) 0
D) −32

Explanation:

- Roots: $z = (2 \pm \sqrt{(4-8)})/2 = 1 \pm i$.
- $z_1 = 1+i$, $z_2 = 1-i$. In polar form: $r = \sqrt{2}$, $\arg(z_1) = \pi/4$.
- $z_1^4 = (\sqrt{2})^4 \cdot e^{\wedge}(i\pi) = 4\cdot(-1) = -4$.
- $z_2^4 = (\sqrt{2})^4 \cdot e^{\wedge}(-i\pi) = 4\cdot(-1) = -4$.
- $z_1^4 - z_2^4 = -4 - (-4) = 0$.
- $a_4 = 0^2 = 0$.
- Answer: C

Question 47
Suppose that $a_n = 2^{n+1} - 3n$, $n \geq 1$. Let S_n be the sum of the first n terms. Then S_n =

A) $2^{n+2} - 3n(n+1)/2 + 4$
B) $2^{n+2} - 3n - 4$
C) $2^{n+2} + 3n(n+1)/2 - 4$
D) $2^{n+2} - 3n(n+1)/2 - 4$

Explanation:

- $S_n = \sum_{k=1}^{n} (2^{k+1} - 3k) = \sum 2^{k+1} - 3\cdot\sum k$.
- $\sum_{k=1}^{n} 2^{k+1} = 2^2 + 2^3 + \ldots + 2^{n+1} = 4\cdot(2^n - 1)/(2-1) = 2^{n+2} - 4$.

- $3\cdot\sum_{k=1}^{n} k = 3\cdot n(n+1)/2$.
- $S_n = 2^{n+2} - 4 - 3n(n+1)/2 = 2^{n+2} - 3n(n+1)/2 - 4$.
- Answer: D

Question 48

A bag contains five balls identical in size and shape, including two red balls, labeled 1 and 2, two black balls, labeled 1 and 2, one white ball, labeled 1. Two balls are drawn randomly without replacement. The probability that the balls have different colors, and the probability that at least one of the two balls is labeled 1 are ( ) respectively.

A) 4/5, 9/10

B) 3/5, 7/10

C) 3/5, 9/10

D) 4/5, 7/10

Explanation:

- Balls: R1, R2 (red), B1, B2 (black), W1 (white). Total outcomes: C(5,2) = 10.
- Different colors: exclude same-color pairs: {R1,R2} and {B1,B2} — 2 pairs.
- Different-color pairs: 10 − 2 = 8. P(different colors) = 8/10 = 4/5.
- At least one labeled 1: complement = both without '1' → only {R2, B2} = 1 pair.
- P(at least one '1') = 1 − 1/10 = 9/10.
- Answer: A (4/5, 9/10)
