CSCA MATHEMATICS EXAMINATION

Wild Mock – March 2026

INSTRUCTIONS:

- Duration: 60 minutes
- Total Score: 100 points
- Total Questions: 48 (Multiple Choice – Single Answer)
- Each question has only ONE correct answer

1. Let $U = \mathbb{R}$ and define $A = \{x \mid x^2 - 2x - 8 < 0\}$ and $B = \{x \mid |x - 1| \geq 3\}$. Find $A \cap (\complement_U B)$.

(A) $(-2, 1)$
(B) $[-2, -2]$
(C) $(-2, -2)$
(D) $\varnothing$ (empty set)

2. Solve the compound inequality system: $\{x^2 - 5x + 4 \leq 0\} \cap \{x / (x^2 - 9) > 0\}$. Express in interval notation.

(A) $[1, 3) \cup (3, 4]$
(B) $(1, 3)$
(C) $[1, 3]$
(D) $(3, 4]$

3. Find all real x satisfying $|2x - 3| + |x + 1| < 4x - 2$.

(A) $(1, +\infty)$
(B) $(3/2, +\infty)$
(C) $(2, +\infty)$
(D) $(5/3, +\infty)$

4. The solution set of the rational inequality $(x^2 - 4x + 3) / (x^2 - x - 6) \geq 0$ is:

(A) $(-\infty, -2) \cup [1, 3] \cup (3, +\infty)$
(B) $(-\infty, -2) \cup [1, 3)$
(C) $[-2, 1] \cup (3, +\infty)$
(D) $(-\infty, -2) \cup (1, 3) \cup (3, +\infty)$

5. Let $A = \{x \mid \log_2(x^2 - 2x) > 0\}$ and $B = \{x \mid 2^{-1} \leq 2^x \leq 4\}$. Find $A \cup B$.

(A) $(-\infty, 0) \cup (2, +\infty)$
(B) $[-1, 0) \cup (2, 3]$
(C) $(-\infty, -1) \cup (0, 2) \cup (2, +\infty)$
(D) $[-1, 3]$

6. The function $f(x) = (1/3)x^3 + ax^2 + bx + c$ is both odd and satisfies $f'(1) = 0$. Determine the value of a.

(A) $a = 0$

(B) $a = -1$
(C) $a = 1$
(D) $a = -1/3$

7. Let $f(x) = x^3 - 3kx$ be monotonically increasing on $(-\infty, -1] \cup [1, +\infty)$. Find the range of k.

(A) $k \le -1$
(B) $k \ge 1$
(C) $k \le 1$
(D) $-1 \le k \le 0$

8. Which function is even AND monotonically decreasing on $(0, +\infty)$ AND has a range that is a proper subset of $[-1, 1]$?

(A) $f(x) = \cos x$
(B) $f(x) = -x^4 + x^2$
(C) $f(x) = 1 - 2x^2$
(D) $f(x) = -|x| / (|x| + 1)$

9. Given that $2f(x) + f(-1/x) = 3x$ for all $x \ne 0$, find $f(2)$.

(A) 13/9
(B) 7/3
(C) 5/3
(D) 11/9

10. Find the inverse function of $f(x) = \ln(x + \sqrt{x^2 + 1})$, $x \in \mathbb{R}$.

(A) $f^{-1}(x) = (e^x - e^{-x}) / 2$
(B) $f^{-1}(x) = e^x + e^{-x}$
(C) $f^{-1}(x) = \sqrt{e^x - 1}$
(D) $f^{-1}(x) = (e^x + e^{-x}) / 2$

11. Let $f: \mathbb{R} \to \mathbb{R}$ satisfy $f(x + y) = f(x) \cdot f(y)$ for all $x, y \in \mathbb{R}$, and $f(1) = 2$. If f is monotonically increasing, find the range of x satisfying $f(2x - 1) < 16$.

(A) $x < 5/2$
(B) $x < 3$
(C) $x < 2$
(D) $x < 7/2$

12. If $\log_3(2) = a$ and $\log_3(5) = b$, express $\log_{60}(450)$ in terms of a and b.

(A) $2a + b + 1 - a \cdot b$
(B) $(2a + b + 1) / (a + b + 1)$
(C) $a + 2b + 1$
(D) $(a + 2b) / (1 + a + b)$

13. Solve for x: $3^{(x+1)} - 3^x - 24 = 0$. Express your answer exactly.

(A) $x = 2$
(B) $x = \log_3(12)$
(C) $x = 3$
(D) $x = \log_3(4)$

14. Given $\log_2(a) = 3 + \log_4(b)$ and $\log_4(a) = 3 + \log_4(b^2)$, find the value of a.

(A) $a = 64$
(B) $a = 16$
(C) $a = 128$
(D) $a = 32$

15. Let $a = 1.2^{0.3}$, $b = 0.3^{1.2}$, $c = \log_{1.2}(0.3)$. Arrange a, b, c in DESCENDING order.

(A) $a > c > b$
(B) $c > a > b$
(C) $a > b > c$
(D) $b > a > c$

16. Given $\sin(\alpha) + \cos(\alpha) = \sqrt{2}/2$, and $\alpha \in (0, \pi)$, find $\tan(\alpha)$.

(A) $-1 + \sqrt{2}$
(B) $-1 - \sqrt{2}$
(C) -1
(D) $1 - \sqrt{2}$

17. Given $\cos(\alpha) = \sqrt{5}/5$ and $\alpha \in (3\pi/2, 2\pi)$, find $\sin(\alpha/2)$.

(A) $-\sqrt{(5\sqrt{5}-5)} / 10$
(B) $\sqrt{((5-\sqrt{5})/10)}$
(C) $-\sqrt{((5-\sqrt{5})/10)}$
(D) $\sqrt{((5+\sqrt{5})/10)}$

18. Simplify: $\sin(3\pi/2 - \alpha)\cdot\cos(2\pi + \alpha) + \cos(3\pi/2 + \alpha)\cdot\sin(\pi - \alpha)$.

(A) -1
(B) 0
(C) 1
(D) $\sin(2\alpha)$

19. If $\tan(\alpha + \beta) = 3/4$ and $\tan(\beta) = 1/7$, find $\tan(2\alpha + 2\beta - 2\beta) = \tan(2\alpha)$.

(A) 24/7
(B) 2/3
(C) 7/24
(D) 3/4

20. The function $f(x) = 2\sin(\omega x + \varphi)\cdot\cos(\omega x + \varphi) + 2\cos^2(\omega x + \varphi) - 1$ has minimum positive period $T = \pi/2$. Given $f(0) = 1$, find ω and φ (with $\omega > 0$, $|\varphi| < \pi/2$).

(A) $\omega = 2$, $\varphi = \pi/8$
(B) $\omega = 4$, $\varphi = \pi/8$
(C) $\omega = 2$, $\varphi = \pi/4$
(D) $\omega = 4$, $\varphi = \pi/4$

21. Solve for $x \in [0, 2\pi]$: $2\sin^2(x/2) - \sqrt{3}\cdot\sin(x) - 1 = 0$.

(A) $x = \pi/3$ or $x = \pi$
(B) $x = 4\pi/3$ or $x = \pi$
(C) $x = \pi/3$ or $x = 4\pi/3$
(D) $x = 2\pi/3$ or $x = \pi$

22. Given $\cos(\alpha - \beta) = \sqrt{2}/2$ and $\sin(\alpha)\cdot\sin(\beta) = \sqrt{2}/4$, find $\cos(2\beta - 2\alpha)$.

(A) 0
(B) 1/2
(C) $\sqrt{2}/2$
(D) −1/2

23. In an arithmetic sequence $\{a_n\}$, if S_n denotes the sum of the first n terms, and $S_5/S_{10} = 1/3$, find S_{15}/S_{30}.

(A) 3/13
(B) 1/7
(C) 1/6
(D) 2/11

24. A geometric sequence $\{a_n\}$ satisfies $a_2 + a_3 + a_4 = 28$ and $a_3 + a_4 + a_5 = 56$. Find a_1.

(A) 2
(B) 1
(C) 3
(D) 4

25. The sum of the first n terms of sequence $\{a_n\}$ satisfies $S_n = 2^{n+1} - 3$. Find $a_1 + a_3 + a_5 + a_7 + a_9$.

(A) $2^{10} - 2 = 1022$
(B) $2^9 - 1 = 511$
(C) $3\times(2^{10}-1) = 3069$
(D) $2^{10} - 1 = 1023$

26. Arithmetic sequence $\{a_n\}$ has $a_1 = 2$ and common difference $d > 0$. If a_1, a_2, a_5 form a geometric sequence, find the sum of the first 100 terms.

(A) 10200
(B) 5250

(C) 10100
(D) 9900

27. An arithmetic sequence $\{a_n\}$ satisfies $3(a_2 + a_6) + 2(a_6 + a_{10} + a_{14}) = 24$. Find S_{13}.
(A) 26
(B) 39
(C) 52
(D) 13

28. Find the equation of the tangent line to the curve $y = x^3 - 3x^2 + 2$ that is parallel to the line $y = -9x + 7$.
(A) $y = -9x - 7$ and $y = -9x + 25$
(B) $y = -9x + 2$ and $y = -9x - 2$
(C) $y = -9x + 25$ only
(D) $y = -9x - 7$ only

29. The function $f(x) = x^3 + ax^2 + bx + 8$ has a local maximum of 8 at $x = 0$ and a local minimum at $x = 2$. Find $a + b$.
(A) -3
(B) 3
(C) 0
(D) -6

30. Find the maximum value of $f(x) = xe^{-x}$ on the interval $[0, 3]$.
(A) $1/e$
(B) $3e^{-3}$
(C) e^{-1}
(D) $2e^{-2}$

31. Given $f(x)$ satisfies $x^2f'(x) + 2xf(x) = \sin(x)$ for all $x \neq 0$, and $f(\pi) = 0$, find $f(\pi/2)$.
(A) $4/\pi$
(B) $2/\pi$
(C) $-4/\pi$
(D) $-2/\pi$

32. The maximum value of $f(x) = -2x^3 + 3x^2 + 12x - 1$ on $[-2, 3]$ is:
(A) 18
(B) 20
(C) 15
(D) 22

33. Given vectors $\vec{a} = (\cos\theta, \sin\theta)$ and $\vec{b} = (1, -\sqrt{3})$, if $\vec{a} \perp (\vec{a} - \vec{b})$, find the value of $\tan(2\theta)$.

(A) $-\sqrt{3}$
(B) $\sqrt{3}$
(C) $1/\sqrt{3}$
(D) $-1/\sqrt{3}$

34. Vectors $\vec{a} = (1, 2)$ and $\vec{b} = (m, 1)$. The projection of $\vec{a}$ onto $\vec{b}$ equals $\sqrt{5}/5$. Find m.
(A) $m = 2$
(B) $m = -1/2$
(C) $m = -2$
(D) $m = 1/2$

35. Given $|\vec{a}| = 2$, $|\vec{b}| = \sqrt{2}$, and the angle between $\vec{a}$ and $\vec{b}$ is 45°. Find $|\vec{a} + 2\vec{b}|^2 - |2\vec{a} - \vec{b}|^2$.
(A) 12
(B) −8
(C) 0
(D) 16

36. Let $z_1 = (1 + i)^2 / (1 - i)$ and $z_2 = (2 + i\sqrt{3})^2$. Find $|z_1 + z_2|$.
(A) $\sqrt{(12 + 4\sqrt{3})}$
(B) $\sqrt{(17 + 4\sqrt{3})}$
(C) $4 + 2\sqrt{3}$
(D) $\sqrt{(13)}$

37. If complex number z satisfies $z^2 - 2z + 4 = 0$, find $z^3 + \bar{z}^3$.
(A) −16
(B) −2
(C) 16
(D) 2

38. On the complex plane, the locus of points z satisfying $|z - 1 - i| = |z + i|$ is a:
(A) Straight line: $x + y = 0$
(B) Parabola
(C) Circle with center (0, 1/2)
(D) Straight line: $x - y + 1 = 0$

39. A circle C passes through the points A(1, 0), B(0, 1), and has its center on the line $y = 2x$. Find the equation of circle C.
(A) $(x - 1)^2 + (y - 2)^2 = 4$
(B) $(x + 1)^2 + (y + 2)^2 = 6$
(C) $(x - 1)^2 + (y - 2)^2 = 5$
(D) $(x - 2)^2 + (y - 4)^2 = 10$

40. Two tangent lines from the external point P(3, 1) are drawn to the circle $x^2 + y^2 = 5$. Find the equation of the chord of contact (polar line).

(A) $3x + y = 5$
(B) $3x - y = 5$
(C) $x + 3y = 5$
(D) $3x + y + 5 = 0$

41. The ellipse $x^2/a^2 + y^2/b^2 = 1$ $(a > b > 0)$ has eccentricity $\sqrt{3}/2$ and passes through A(1, $\sqrt{3}/2$). Find the value of $a^2 - b^2$.

(A) 3
(B) 1
(C) 3/4
(D) 4

42. A hyperbola has the same foci as the ellipse $x^2/16 + y^2/4 = 1$, and passes through (2, 0). Find the asymptotes of the hyperbola.

(A) $y = \pm\sqrt{2} \cdot x$
(B) $y = \pm x$
(C) $y = \pm(1/\sqrt{2})\, x$
(D) $y = \pm(\sqrt{3}/3)\, x$

43. The directrix of the parabola $y^2 = 4px$ $(p > 0)$ passes through the focus of the ellipse $x^2/5 + y^2/4 = 1$ (left focus). Find p.

(A) $p = 1$
(B) $p = 2$
(C) $p = 1/2$
(D) $p = 4$

44. Given two straight lines l_1: $ax + by + 1 = 0$ and l_2: $(a + 2)x + (b - 3)y + 1 = 0$. If $l_1 \parallel l_2$, find all possible values of $3a - 2b$.

(A) −6
(B) 6 or −6
(C) 6
(D) 0

45. A bag contains 5 red balls and 3 blue balls. Two balls are drawn without replacement. Given that at least one ball is red, find the probability that both balls are red.

(A) 5/14
(B) 5/7
(C) 10/21
(D) 2/7

46. In a city, records show that on any given day: P(rain) = 0.4, and if it rains, P(traffic jam) = 0.7; if it does not rain, P(traffic jam) = 0.3. On a randomly chosen day, given that there IS a traffic jam, find the probability that it rained. Use Bayes' theorem.

(A) 14/23
(B) 7/10
(C) 28/46
(D) 7/13

47. Suppose $a_1 = 1$ and $a_{n+1} = a_n / (2a_n + 1)$ for $n \geq 1$. Define $b_n = 1/a_n$. Find the general formula for b_n, and hence a_{100}.

(A) $a_{100} = 1/199$
(B) $a_{100} = 1/201$
(C) $a_{100} = 1/101$
(D) $a_{100} = 1/99$

48. The Chinese zodiac (Shēngxiào) follows a 12-year cycle: Rat, Ox, Tiger, Rabbit, Dragon, Snake, Horse, Goat, Monkey, Rooster, Dog, and Pig. A researcher selects a group of 4 people whose birth years form a strictly increasing arithmetic sequence with a common difference d = 4. If the first person was born in a random year between 1980 and 2003 inclusive, what is the probability that at least two people in this group share the same zodiac sign?

(A) 1
(B) 1/2
(C) 2/3
(D) 0

END OF EXAMINATION